Math for S1P and S2P Files

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What is this about?

S21 to Z = R+jX and v.v. ('S2Z' and 'Z2S') for Hi-Z values

S21 to Z = R+jX (S2Z) for low-Z values

S11 to Z = R+jX and v.v. (S2Z and Z2S)

Zs = Rs + jXs calculating series to parallel Rp and jXp and v.v.

Things you should know about the S1P and S2P files

The file-format that nanoSAVER uses is a standard format and contains several columns:

S1P-file (3 col): Stim(ulus) Re(S11)   Im(S11)

S2P-file (9 col): Stim(ulus) Re(S11)   Im(S11) Re(S21)   Im(S21) Re(S12)   Im(S12) Re(S22)   Im(S22)

Stimulus is the used frequency and the rest of the columns contain the S-values. The S-values are split-up in 'Re' and 'Im'. 'Re' is the so called real part of the S-value and 'Im' is the imaginary part. With this we do not only know the amplitude of the measured signal but also the phase. The nanoVNA does not measure S12 and S22, because then it would need a signal source in the second port (CH1) and that is not the case. NanoSAVER will therefore export zeros in those S2P columns.

An S21-file imported into Excel looks like this:

	A	B	C	D	E	F	G	H	I
1	#	Hz	S	RI	R	50			
2	500000	0,317827	-5,33E-05	0,680673	-0,00019	0	0	0	0
3	795000	0,317765	0,000451	0,680574	-0,0002	0	0	0	0
4	1090000	0,317748	0,000244	0,680465	-0,00021	0	0	0	0

Attention: the first row does not contain the column-names! It tells us something about the file itself:

- Hz: shown frequency is in Hz
- S: the file contains S-parameters
- RI: the file is in the Re(Sxx) Im(Sxx) format, so S is split in real part and imaginary part.
- 50: characteristic impedance of the measurement system (50 ohm).

So for this file we've got:
frequency (Hz), Re(S11), Im(S11), Re(S21), Im(S21) and the already mentioned zeros, because S12 and S22 are not measured here.

In the next paragraphs Re(Sxx) and Im(Sxx) are expressed as Dr (r = real) and Di (i = imaginary).

(Disclaimer: For the 'purist' readers: Attenuation expressed in positive dB's is same as Gain expressed in negative dB's.   ;-))

S21 to Z = R+jX and v.v. (S2Z and Z2S) for Hi-Z measurement values

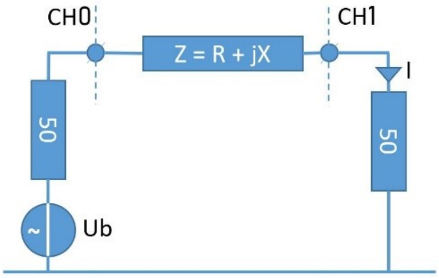
S2Z: from Attenuation (or Gain) in the nanoVNA S21 to Z = R + jX

Calibration current with Short (Z=0) : $I_0 = \frac{Ub}{100}$

With a load Z: $I_Z = \frac{Ub}{100+R+jX}$ (smaller current)

S21 = Re(S11) + j Im(S11) = measure for attenuation D = Dr + j Di

$Dr + j\,Di = \frac{Iz}{I0} = \frac{100}{R + 100 + jX} = \frac{100}{Rt + jX}$ (Rt = R + 100)



$$Rt + jX = \frac{100}{Dr + jDi}$$

$$Rt + jX = \frac{100(Dr - jDi)}{Dr^2 + Di^2}$$

Split up the real part and the imaginary part

R	X
$Rt = \frac{100Dr}{Dr^2 + Di^2}$ $R = \frac{100Dr}{Dr^2 + Di^2} - 100$	$X = -\frac{100\mathrm{Di}}{Dr^2 + Di^2}$

Z2S: from Z = R+jX back to S21

$$Dr + jDi = \frac{100}{Rt + jX}$$

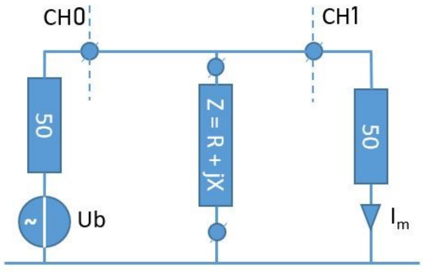
$$Dr + jDi = \frac{100Rt - 100jX}{Rt^2 + X^2}$$

Split up the real part and the imaginary part

Dr = Re(S21)	X = Im(S21)
$Dr = \frac{100Rt}{Rt^2 + X^2}$ $Dr = \frac{100(\mathrm{R} + 100)}{(R + 100)^2 + X^2}$	$Di = -\frac{100X}{Rt^2 + X^2}$ $Di = -\frac{100X}{(R + 100)^2 + X^2}$

S21 to Z = R+jX (S2Z) for low-Z values

S2Z: from attunuation (or Gain) in the nanoVNA S21 to Z = R + jX where Z is in parallel to CH1.



Calibration without Z connected: $I_0 = \frac{Ub}{100}$

If we connect Z in parallel then:

$$Im = \frac{Z}{Z+50} * \frac{Ub}{50 + \left(\frac{50Z}{Z+50}\right)}$$

$$Im = Ub * \frac{Z}{100Z + 2500}$$

Attenuation D (= S21) = $Dr + jDi = \frac{Im}{Io}$

$$D = \left(\frac{\frac{Z}{100Z+2500}}{\frac{1}{100}}\right) = \frac{Z}{Z+25}$$

$$ZD + 25D - Z = 0$$

$$Z = \frac{25D}{1-D}$$

$$Z = R + jX = \frac{25(Dr + jDi)}{1 - Dr - jDi}$$

$$R + jX = \frac{25(Dr + jDi)(1 - Dr + jDi)}{(1-Dr)^2 + Di^2}$$

$$R + jX = 25 * \frac{Dr - Dr^2 + jDrDi + jDi - JDrDi - Di^2}{(1-Dr)^2 + Di^2} = 25 * \frac{(Dr - Dr^2 - Di^2) + j(Di)}{(1-Dr)^2 + Di^2}$$

Split up the real and imaginary parts of this equation

$R = 25 * \frac{Dr(1-Dr) - Di^2}{(1-Dr)^2 + Di^2}$	$X = 25 * \frac{Di}{(1-Dr)^2 + Di^2}$

S11 to Z = R+jX and v.v. (S2Z and Z2S)

Z2S:

$$S11 = \frac{Z - Zo}{Z + Zo}$$

$$S11 = Sr + jSi = \frac{R + jX - 50}{R + jX + 50}$$

$$Sr + jSi = \frac{(R - 50) + jX}{(R + 50) + jX}$$

$$Sr + jSi = \frac{[(R + 50) - jX][(R - 50) + jX]}{(R + 50)^2 + X^2}$$

$$Sr + jSi = \frac{(R + 50)(R - 50) + X^2}{(R + 50)^2 + X^2} + \frac{100jX}{(R + 50)^2 + X^2}$$

$Sr = \frac{R^2 - 2500 + X^2}{(R + 50)^2 + X^2}$	$Si = \frac{100X}{(R + 50)^2 + X^2}$

S2Z:

$$\frac{Z}{Zo} = \frac{1 + S11}{1 - S11}$$

$$Z = 50\left(\frac{1 + Sr + jSi}{1 - Sr - jSi}\right)$$

$$R + jX = 50\frac{\left((1 + Sr + jSi)(1 - Sr + jSi)\right)}{(1 - Sr)^2 + Si^2}$$

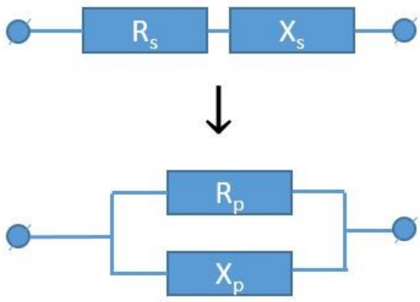
$$R + jX = 50\frac{1 - Sr + jSi - Sr^2 + Sr + jSiSr + jSi - jSiSr - Si^2}{(1 - Sr)^2 + Si^2}$$

$$R + jX = 50\left(\frac{1 - (Si^2 + Sr^2) + 2jSi}{(1 - Sr)^2 + Si^2}\right)$$

$Rs = 50\frac{1 - (Si^2 + Sr^2)}{(1 - Sr)^2 + Si^2}$	$Xs = \frac{100Si}{(1 - Sr)^2 + Si^2}$

Zs = Rs + jXs calculation to Rp and jXp and v.v.

$$Zs = Rs + jXs$$



$$Ys = \frac{1}{Rs + jXs}$$

$$Ys = \frac{Rs - jXs}{Rs^2 + Xs^2}$$

$$Ys = \frac{Rs}{Rs^2 + Ys^2} - \frac{jXs}{Rs^2 + Xs^2}$$

$$Yp = \frac{1}{Rp} + \frac{1}{jXp}$$

$Rp = \frac{Rs^2 + Xs^2}{Rs}$	$jXp = \frac{Rs^2 + Xs^2}{-jXs}$
$Rp = \frac{Xs^2}{Rs} + Rs$	$Xp = \frac{Rs^2}{Xs} + Xs$

$$Zp = \frac{jRpXp}{Rp + jXp}$$

$$Zp = \frac{jRpXp(Rp - jXp)}{Rp^2 + Xp^2}$$

$$Zp = \frac{RXp^2}{Rp^2 + Xp^2} + \frac{jRp^2Xp}{Rp^2 + Xp^2}$$

$$Zs = Rs + jXs$$

$Rs = \frac{RpXp^2}{Rp^2 + Xp^2}$	$Xs = \frac{Rp^2Xp}{Rp^2 + Xp^2}$

73,

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July 2021